perspective

Majorana returns

Frank Wilczek

In his short career, Ettore Majorana made several profound contributions. One of them, his concept of 'Majorana fermions' — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

Enrico Fermi had to cajole his friend Ettore Majorana into publishing his big idea: a modification of the Dirac equation that would have profound ramifications for particle physics. Shortly afterwards, in 1938, Majorana mysteriously disappeared, and for 70 years his modified equation remained a rather obscure footnote in theoretical physics (Box 1). Now suddenly, it seems, Majorana's concept is ubiquitous, and his equation is central to recent work not only in neutrino physics, supersymmetry and dark matter, but also on some exotic states of ordinary matter.

Majorana fermions

An electrically charged particle is different from its antiparticle as it has the opposite electric charge, and electric charge is a measurable, stable property. It is possible, however, for an electrically neutral particle to be its own antiparticle. Photons, which have spin 1 in units of the rationalized Planck's constant $\hbar$, are a familiar case; neutral pions (spin 0) are a further example, and gravitons (spin 2) another. Particles that are their own antiparticles must be created by fields φ that obey $\varphi = \varphi^*$ — that is, real fields, because the complex-conjugate fields φ^* create their antiparticles. The equations for particles with spin 0, spin 1 and spin 2 — the Klein–Gordon, Maxwell (electromagnetism) and Einstein (general relativity) equations, respectively — readily accommodate real fields, as these equations are formulated using real numbers.

On the other hand, the neutron (which has spin ½), despite being electrically neutral, is not its own antiparticle: several neutrons can peacefully coexist within an atomic nucleus, but an antineutron rapidly annihilates. Neither, of course, are the most famous spin-½ particles — electrons and protons, which are electrically charged — their own antiparticles. So it is not obvious that we need an equation to describe spin-½ particles that are their own antiparticles.

Indeed, when, in 1928, Paul Dirac discovered[1] the theoretical framework for describing spin-½ particles, it seemed that complex numbers were unavoidable (Box 2). Dirac's original equation contained both real and imaginary numbers, and therefore it can only pertain to complex fields. For Dirac, who was concerned with describing electrons, this feature posed no problem, and even came to seem an advantage because it 'explained' why positrons, the antiparticles of electrons, exist.

Enter Ettore Majorana. In his 1937 paper[2], Majorana posed, and answered, the question of whether equations for spin-½ fields must necessarily, like Dirac's original equation, involve complex numbers. Considerations of mathematical elegance and symmetry both motivated and guided his investigation. Majorana discovered that, to the contrary, there is a simple, clever modification of Dirac's equation that involves only real numbers. With this discovery, Majorana made the idea that spin-½ particles could be their own antiparticles theoretically respectable, that is, consistent with the general principles of relativity and quantum theory. In his honour, we call such hypothetical particles Majorana fermions. But are there physical examples?

Are neutrinos Majorana fermions?

Majorana speculated that his equation might apply to neutrinos. In 1937, neutrinos were themselves hypothetical, and their properties unknown. The experimental study of neutrinos commenced with their discovery[3] in 1956, but their observed properties seemed to disfavour Majorana's idea. Specifically, there seemed to be a strict distinction between neutrinos and antineutrinos.

The distinction is connected with the law of lepton-number conservation, which applies for each of the leptons — electron (e), muon (μ) and tau (τ). For example, for electrons, lepton-number conservation means that, in any reaction, the total number of electrons minus the number of antielectrons, plus the number of electron neutrinos minus the number of antielectron neutrinos is a constant (call it L_e). These laws lead to many successful selection rules. For example, the particles (muon neutrinos, ν_μ) emitted in positive pion (π) decay, $\pi^+ \rightarrow \mu^+ + \nu_\mu$, will induce neutron-to-proton conversion $\nu_\mu + n \rightarrow \mu^- + p$, but not proton-to-neutron conversion $\nu_\mu + p \rightarrow \mu^+ + n$; the particles (muon antineutrinos, $\bar{\nu}_\mu$) emitted in the negative pion decay $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ obey the opposite pattern. Indeed, it was through studies of this kind that the existence of different 'flavours' of neutrino, corresponding to the different types of charged lepton was discovered[4].

Of course, if neutrinos really differ from antineutrinos, then they are not Majorana fermions. In recent years, however, the situation has come to seem less clear-cut, for it has been discovered that neutrinos oscillate in flavour[5]. For example, an electron antineutrino emitted from the Sun can arrive at Earth as a muon antineutrino or a tau antineutrino. In some sense this is a small effect, but when neutrinos travel a long way they have time to do rare things. These flavour oscillations show that the separate 'laws' of lepton-number conservation do not hold: at best, only the sum $L_e + L_\mu + L_\tau$ can be strictly conserved.

Thus awakened from our dogmatic slumber, we re-open Majorana's question: could the distinction between neutrino and antineutrino, which seems so plainly apparent, be superficial? (Consider the vast perceptual disconnect between the morning star and the evening star — yet they're both Venus.)

But how can $\nu = \bar{\nu}$ be reconciled with those many observations that seemed to indicate a distinction? The point is that the ν particles produced in, for example, $\pi^+ \rightarrow \mu^+ + \nu$ are in a very different state of motion from the $\bar{\nu}$ particles produced in $\pi^- \rightarrow \mu^- + \bar{\nu}$. The former are left handed, spinning in the sense that the fingers of your left hand point, if your thumb aligns with the velocity, whereas the latter are right handed. So, logically, ν and $\bar{\nu}$ and might be the same particle, having different behaviours when it is in different states of motion.

If you could bring neutrinos and antineutrinos to rest, and do experiments with them, you could test whether they behave the same way. That is impractical, unfortunately: theoretically, the cosmos is awash with slow neutrinos, but they are too hard to detect. Although such a direct test of Majorana's hypothesis seems out of reach for now, several ambitious experiments are underway to test one of its implications, namely, that even the last bastion of lepton-number conservation, $L_e + L_\mu + L_\tau$, can be toppled. Searches for neutrino-less double β decay, such as $Ge^{76} \rightarrow Se^{76} + 2e$, are launching a promising fusillade[6]. In this decay, total lepton number changes by two, so its occurrence would disprove the conservation law definitively.

Meanwhile, the leading ideas on neutrino masses, rooted in unified field theories, predict that neutrinos are Majorana fermions[7,8]. The detailed logic is complex, but the basic idea is simple: we get more economical, and much prettier, equations if we don't add antineutrinos as separate entities to our fundamental theory. For if neutrinos in the right-handed state of motion are not antineutrinos, they must be something else; and that something else must (as it's escaped detection so far) interact with the kinds of matter we know very feebly indeed. It is hard to fit such oddball entities within the most attractive unified theories, which require symmetry among their building blocks.

Of supersymmetry and dark matter

Neutrinos were Majorana's own candidates for Majorana fermions, and although they look more promising than ever in that regard, no longer are they unique. Other problems at the frontier of fundamental physics seem to call for more Majorana fermions.

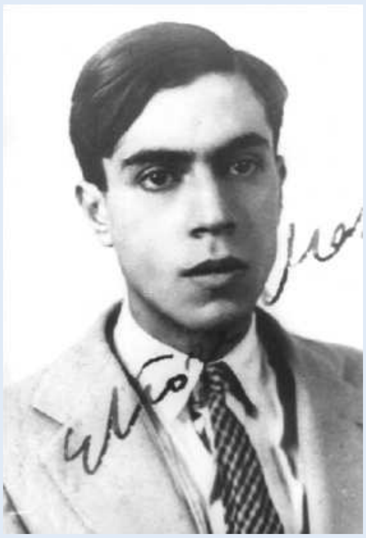
Supersymmetry is a leading proposal to improve the symmetry and coherence of the equations of physics[9]. It involves the expansion of spacetime into a new, quantum dimension. Particles that move in that direction change their mass and spin. If supersymmetry is valid, then every known bosonic (integer spin) particle will have a heavier fermionic (half-integer spin) partner; and vice versa for each known fermion. There is suggestive, although circumstantial, evidence for the existence of these 'superpartners'. Specifically, if the superpartners exist and are not too heavy, then in their evanescent form, as virtual particles, they are computed to modify (partially screen) the basic units of strong, weak and electromagnetic charge so as to quantitatively account for the different observed charge values — in a unified field theory where, fundamentally, those values are equal[10]. In brief, supersymmetry allows the unification of the fundamental forces.

If supersymmetry is valid, then the photon has as its superpartner a spin-½ particle, the photino. As the photino mirrors the properties of the photon, it must be its own antiparticle. Thus the photino is a Majorana fermion. So, for similar reasons, are various other superpartners (such as neutral gauginos, as well as Higgsinos). In a word, supersymmetry comes chock-a-block with Majorana fermions. If, as widely anticipated, superpartners are produced — as real, not just virtual, particles — at the Large Hadron Collider, we might quickly

Box 1 | The romance of Ettore Majorana

"There are many categories of scientists: people of second and third rank, who do their best, but do not go very far; there are also people of first-class rank, who make great discoveries, fundamental to the development of science. But then there are the geniuses, like Galileo and Newton. Well Ettore Majorana was one of them." Enrico Fermi, not known for flightiness or overstatement, is the source of these much-quoted lines.

The bare facts of Majorana's life are briefly told. Born in Catania, Italy, on 5 August 1906, into an accomplished family, he rose rapidly through the academic ranks, became a friend and scientific collaborator of Fermi, Werner Heisenberg and other luminaries, and produced a stream of high-quality papers. Then, beginning in 1933, things started to go terribly wrong. He complained of gastritis, became reclusive, with no official position, and published nothing for several years. In 1937, he allowed Fermi to write-up and submit, under his (Majorana's) name, his last and most profound paper — the point of departure of this article — containing results he had derived some years before. At Fermi's urging, Majorana applied for professorships and was awarded the Chair in Theoretical Physics at Naples,



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which he took up in January 1938. Two months later, he embarked on a mysterious trip to Palermo, arrived, then boarded a ship straight back to Naples and disappeared without a trace.

Majorana published only nine papers in his lifetime, none very lengthy. They are collected, with commentaries, all in both Italian and English versions, in a slim volume[30]. Each is a substantial contribution to quantum physics. At least two are masterpieces: the last, as mentioned, and another on the quantum theory of spins in magnetic fields, which anticipates the later brilliant development of molecular-beam and magnetic resonance techniques.

In recent years, a small industry has developed, bringing Majorana's unpublished notebooks into print (see for example ref. 31). They are impressive documents, full of original calculations and expositions covering a wide range of physical problems. They leave an overwhelming impression of gathering strength; physics might have advanced more rapidly on several fronts had Majorana pulled this material together and shared it with the world.

How did he vanish? There are two leading theories. According to one, he retired to a monastery, to escape a spiritual crisis and accept the embrace of his deep Catholic faith (not unlike another tortured scientific genius, Blaise Pascal). According to another, he jumped overboard, an act of suicide recalling the alienated supermind of fiction, Odd John[32]. Fermi's appreciation had a wistful conclusion, which is less well known: "Majorana had greater gifts than anyone else in the world. Unfortunately he lacked one quality which other men generally have: plain common sense."

Box 2 | The Majorana equation

In 1928, Dirac proposed his relativistic wave equation for electrons[33]. This was a watershed event in theoretical physics, leading to a new understanding of spin, predicting the existence of antimatter, and impelling — for its adequate interpretation — the creation of quantum field theory. It also inaugurated a new method in theoretical physics, emphasizing mathematical aesthetics as a source of inspiration. Majorana's most influential work is especially poetic, in that it applies Dirac's method to Dirac's equation itself, to distill from it an equation both elegant and new. For many years, Majorana's idea seemed to be an ingenious but unfulfilled speculation. Recently, however, it has come into its own, and now occupies a central place in several of the most vibrant frontiers of modern physics.

Dirac's equation connects the four components of a field ψ. In modern (covariant) notation it reads

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$$

The γ matrices are required to obey the rules of Clifford algebra, that is

$$\{\gamma^{\mu}\gamma^{\nu}\} \equiv \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}$$

where $\eta^{\mu\nu}$ is the metric tensor of flat space. Spelling it out, we have

$$(\gamma^0)^2 = -(\gamma^1)^2 = -(\gamma^2)^2 = -(\gamma^3)^2 = 1$$
$$\gamma^j\gamma^k = -\gamma^k\gamma^j \text{ for } i \neq j$$

(in which I have adopted units such that $\hbar = c = 1$). Furthermore, we require that γ^0 be Hermitian, and the remaining marices anti-Hermitian. These conditions ensure that the equation properly describes the wavefunction of a spin-½ particle with mass m.

Dirac found a suitable set of 4 × 4 γ matrices, whose entries contain both real and imaginary numbers. For the equation to make sense, ψ must then be a complex field. Dirac and most other physicists regarded this consequence as a good feature, because electrons are electrically charged, and the description of charged particles requires complex fields, even at the level of the Schrödinger equation. This is also true in the language of quantum field theory. In quantum field theory, if a given field φ creates the particle A (and destroys its antiparticle $\bar{A}$), the complex conjugate φ^* will create $\bar{A}$ and destroy A. Particles that are their own antiparticles must be associated with fields obeying $\varphi = \varphi^*$, that is, real fields. Because electrons and positrons are distinct, the associated fields ψ and ψ^* and must therefore be different; this feature appeared naturally in Dirac's equation.

Majorana inquired whether it might be possible for a spin-½ particle to be its own antiparticle, by attempting to find the equation that such an object would satisfy. To get an equation of Dirac's type (that is, suitable for spin-½) but capable of governing a real field, requires γ matrices that satisfy the Clifford algebra and are purely imaginary. Majorana found such matrices. Written as tensor products of the usual Pauli matrices σ, they take the form:

$$\tilde{\gamma}^0 = \sigma_2 \otimes \sigma_1$$
$$\tilde{\gamma}^1 = i\sigma_1 \otimes 1$$
$$\tilde{\gamma}^2 = i\sigma_3 \otimes 1$$
$$\tilde{\gamma}^3 = i\sigma_2 \otimes \sigma_2$$

or alternatively, as ordinary matrices:

$$\tilde{\gamma}^0 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^1 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^2 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

$$\tilde{\gamma}^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

Majorana's equation, then, is simply

$$(i\tilde{\gamma}^{\mu}\partial_{\mu} - m)\tilde{\psi} = 0)$$

Because the $\tilde{\gamma}^{\mu}$ matrices are purely imaginary, the matrices $i\tilde{\gamma}^{\mu}$ are real, and consequently this equation can govern a real field $\tilde{\psi}$.

establish the existence of several Majorana fermions, even as the status of neutrinos remains uncertain.

A popular hypothesis[11] for the astronomical dark matter is that it is a weakly interacting massive particle, or WIMP. Indeed, it could be one of the superpartners just mentioned. The overall neutrality of Majorana fermions means that they can decay, or annihilate in pairs. The debris from such events could produce energetic cosmic rays, which are the object of ongoing search experiments. It is entirely possible that WIMPs, dominating the mass of the Universe and proclaiming their existence with cosmic fireworks, will be the first established Majorana fermions.

Majorana modes in the solid state

There is a completely different area of physics in which Majorana's idea is starting to receive more attention — theoretical solid-state physics. Recent investigations suggest that exotic quasiparticle excitations in a variety of interesting condensed-matter systems are Majorana fermions. Many of these ideas were born of high mathematical fantasy, but there is a very real chance that they may soon mature into a surprisingly tangible, and even useful, form.

The concept of excitations that are their own antiparticles is not unprecedented in solid-state physics. An example is the exciton — a quasiparticle formed by bound states of electrons and holes. The latter are a familiar concept in modern solid-state physics[12], and represent the absence of an electron in a mode that is normally (in the overall ground state) occupied. In rough but more vivid language, holes are bubbles of emptiness in the Fermi sea of electrons (Fig. 1a). Holes 'look' and 'behave' like the antiparticles or antimatter to their corresponding particles, the valence electrons; they act as if they were positively charged electrons.

The particle–antiparticle correspondence, as well as the manifestation of the electron's and hole's characteristic fermion statistics, is transparent in the mathematical formalism of second quantization. Here, 'particle states' are associated with creation operators $c_j^{\dagger}$, antiparticle (hole) states with their conjugate operators, c_j. In essence, c_j can create a hole, or destroy a particle, in state j, whereas $c_j^{\dagger}$ can create a particle, or destroy a hole in state j. Three key relations embody the characteristics of Fermi–Dirac statistics and describe the relationship between particle and hole operators associated with different states. First,

$$(c_j^{\dagger})^2 = c_j^2 = 0$$

which means that the attempt to cram two electrons, or two holes, into the same state comes to naught — a manifestation of Pauli's exclusion principle. Second, for two distinct (orthogonal) states j and k,

$$\{c_j^\dagger, c_k\} \equiv c_j^\dagger c_k + c_k c_j^\dagger = \{c_j, c_k\} = \{c_j^\dagger, c_k^\dagger\} = 0$$

which is a consequence of the antisymmetry of Fermi–Dirac statistics. Third,

$$\{c_j^\dagger, c_j\} = \{c_k^\dagger, c_k\} = 1$$

which is the completeness relation.

In this formalism, particle–hole interchange (charge conjugation) is implemented by $c_j \leftrightarrow c_j^\dagger$. Because electrons and holes have opposite charge, they are not their own antiparticles and therefore not Majorana fermions. Excitons, on the other hand, are bound states of electrons and holes, and thus, in the language of second quantization, they are created by combinations of electron and hole operators, of the general form $c_j^\dagger c_k + c_k c_j^\dagger$. Under charge conjugation, this exciton 'creation' operator goes over into itself, and therefore the excitations it creates are their own antiparticles. But conventional excitons are always bosons, with integer spin, and thus can make no call on Majorana's legerdemain. In this sense they are analogous to the photons of conventional particle physics.

Superconductors to the rescue

So can there ever be a solid-state situation in which half-integer-spin particles are their own antiparticles? At first sight it seems hopeless to realize Majorana fermions from the raw material of electrons in solids, simply because electrons are charged, and therefore definitely different from their antimatter counterparts, the (oppositely charged) holes. But superconductivity changes the picture[13], because in superconductors the absolute distinction between electrons and holes is blurred (Fig. 1b,c). In such materials, electrons form so-called Cooper pairs, which, owing to their boson-like nature, can form a dense 'condensate', unimpeded by the Pauli exclusion principle. Indeed, it is just this condensate that, theoretically, is responsible for superconductivity[13].

As a consequence, electron number is in effect no longer conserved: two electrons (in a Cooper pair) can be added or subtracted from the condensate without substantially changing its properties. Crucially too, the superconductor screens electric and confines magnetic fields so that charge is no longer observable (Fig. 1c). Thus, in a superconductor the most daunting barrier to producing Majorana-like excitations — the charge-conjugation hurdle — seems vulnerable.

Indeed, already in the earliest days following Bardeen, Cooper and Schrieffer's triumphant theory of superconductivity (BCS theory)[14], it was realized that certain fermionic modes in the superconducting state are created by mixtures of what were, in the normal state, electron and hole operators. In physical terms, a (normal state) electron mode can lower its energy, in the superconducting state, by mixing with a (normal state) hole mode attached to a Cooper pair. Mathematically, this phenomenon is encoded in the Bogoliubov–Valatin formalism[13]. Therein one finds that the creation operators for modes in the superconducting state are mixtures of electron and hole creation operators, in the form $\cos\theta\, c_j + \sin\theta\, c_k^\dagger$. But electrons in such Bogoliubov–Valatin modes are not exactly their own antiparticles (except accidentally, in the specific case $j = k$ and $\theta = \pm\pi/4$) and thus, such modes are not a realization of Majorana fermions.

However, there are certain types of superconductor in which Majorana-type excitations are predicted to emerge. For instance, some superconductors can contain magnetic flux tubes, also known as Abrikosov vortices[15], the presence of which alters the equations for the electrons. In particular, depending on the kind of superconductor and the electronic spectrum, the vortices may trap so-called zero modes, spin-1/2 'excitons' of very low (formally, zero) energy. The zero modes are discrete; there are a finite number associated with each vortex. The existence of these modes is related to a profound result in mathematics, the Atiyah–Singer index theorem, which connects the existence of special, symmetric solutions of differential equations to the topology of the parameters that appear in those equations[16].

The zero modes are mixtures of particles and holes in equal measure, and thus one can call the quasiparticles associated with these zero modes 'partiholes'. Such partiholes differ crucially from conventional excitons. They are created by operators of the form $\gamma_j = c_j^\dagger + c_j$. As γ_j is left invariant by the charge conjugation, $c \leftrightarrow c^\dagger$, partihole operators create localized spin-½ particles that are their own antiparticles. In this sense, partiholes are a new instance of Majorana's idea, which is why the corresponding zero modes are called Majorana modes.

But where and how would one observe such Majorana modes? In most superconductors, in which the Cooper pairs have orbital angular momentum 0 (s-wave) and the electrons obey a Schrödinger-like, nonrelativistic equation, zero modes are not predicted to occur. However, they are predicted[17] to occur if the Cooper pairs have orbital angular momentum 1 ($p_x + ip_y$-wave), or for s-wave Cooper pairing if the electrons in the normal state obey a Dirac-like equation[18]. The former case could occur, in effect, in certain quantum Hall states — specifically, the so-called Pfaffian or Moore–Read state at $\nu = 5/2$ filling[19] — and possibly in some exotic superconductors including strontium ruthenate[20]. The case in which the normal-state electrons obey Dirac-like beahviour is predicted to be induced at the surface of a new class of material called topological insulators[21] or in graphene[22], by exploiting the proximity effect to induce superconductivity in those materials. Thus, the race is on to find such exotic realizations of Majorana's idea in a variety of systems.

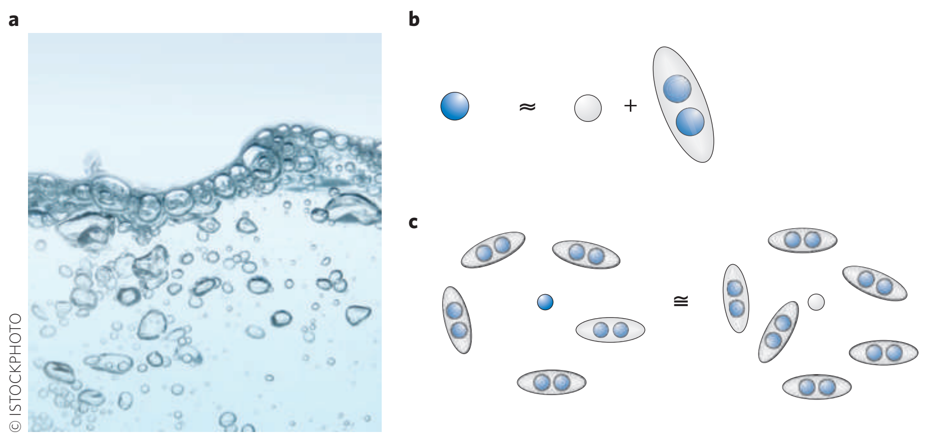

Figure 1 | Antimatter matters in the solid state. **a**, A familiar concept in solid-state physics, holes are bubbles of missing electrons in the Fermi sea of the electronic spectrum, behaving like positively charged electrons. **b**, In a superconductor, the properties of electrons (blue) and holes (grey) are drastically modified by their interaction with the surrounding sea of Cooper pairs; a hole can attract or bind to a Cooper pair, and acquire negative charge. **c**, More importantly, Cooper pairs cluster around holes and thin out around electrons, in such a way that no rigorous distinction between them remains.

Tomorrow's qubits?

Majorana modes open a portal into some extremely unusual and interesting aspects of quantum theory. For instance, Majorana modes on Abrikosov vortices have the unique feature that the operators that create them square to the identity, not to zero — that is, $\gamma_j^2 = 1$. Thus the object created by γ_j is not a conventional fermion. Nor is it a conventional boson. Indeed, adding a second partihole to a state already occupied by one partihole neither annihilates the state nor creates an essentially new, doubly occupied state; rather, it recreates the state of zero occupancy.

To further explain the quantum statistics of Majorana modes, let us consider several of them, and see what happens when they are interchanged. At this point, it is convenient to restrict our consideration to two spatial dimensions, such that the vortices trapping the zero modes reduce to point defects rather than lines. In such two-dimensional settings, the possibility of quantum statistics more general than bosons or fermions — anyons — has been discussed vigorously in recent years[23,24]. The simplest (abelian) anyons can obey statistics ranging continuously between Fermi–Dirac and Bose–Einstein statistics, because the anticlockwise exchange of one anyon around another gives rise to a complex phase, $|\psi_1\psi_2\rangle = e^{i\theta}|\psi_2\psi_1\rangle$, which can be different from ± 1 ($\theta = 2\pi$ for bosons and $\theta = \pi$ for fermions). Anyons are included in the conventional theory of the fractional quantum Hall effect[25], and there are efforts underway to demonstrate their existence experimentally.

Majorana modes have a statistic that is different and more complex than conventional anyons. Specifically, the statistic is inherently non-abelian — exchanges of particles associated with Majorana modes result not only in a change of the phase of the quantum mechanical wavefunction, but also in the change of the internal states of the modes. A mathematical analysis[26], though not especially difficult, is well beyond the scope of this article; but some indications are in order. Separated Majorana partihole operators (of quantity n) obey the algebra $\{\gamma^j, \gamma^k\} = 2\delta_{jk}$.

This is another Clifford algebra, similar to the one in the Dirac or Majorana equations (Box 2). However, in contrast to the algebra associated with the Dirac and Majorana equations, the algebra describing Majorana modes describes the geometry of n Euclidean dimensions in an abstract mode space, rather than the paltry 3+1 dimensions of spacetime. In this mode space, the process of exchanging neighbouring modes characterized by the indices j and k induces the transformation

$$\gamma_j \rightarrow \gamma_k$$
$$\gamma_k \rightarrow -\gamma_j$$

in which the minus sign is all-important. This transformation is just what we would get from a $\pi/2$ rotation in the j–k plane, with γ_i regarded as a vector. The minimal realization of all these transformations is the so-called spinor representation, which is $2^{[(n+1)/2]}$-dimensional[27]. In this way, simple exchange operations in physical space induce complex motions in quantum-mechanical Hilbert space.

This power to evolve simple operations in physical space into complex motions in an exponentially large Hilbert space is thought to provide qualitatively new and powerful methods for quantum information processing[28]. This is the vision of 'topological quantum computing'[29]. Many of the systems mentioned above — such as quantum Hall states, exotic superconductors, surfaces at which conventional superconductors and topological insulators abut — as well as skilfully engineered optical-lattice/cold-atom systems and systems of quantum wires are currently under very active investigation, including through experimental work, as candidate embodiments of that vision.

Whatever the fate of these particular explorations, there is no doubt that Majorana's central idea, which long seemed peripheral, has secured a place at the core of theoretical physics. It would be both disappointing and surprising if real Majorana fermions, now ardently sought, do not soon materialize. ❐

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