LETTERS TO NATURE

Is our vacuum metastable?

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In spontaneously broken gauge theories of particle interactions there are sometimes several local minima of the effective potential. Any of these minima can serve as a vacuum in the sense that we can expand the fields around their values at the minimum, interpret the quantized fluctuations around the minimum as particles, and compare the properties of these particles with experiment. One might think that only the state with absolutely minimum energy could be what we ordinarily call our vacuum, as the other local minima will inevitably decay into this lowest one. However, this is not necessarily the case, because it is possible for the lifetime of a metastable vacuum to be very long, even when compared with the age of the Universe. Furthermore, the energy densities available in present laboratory or astrophysical environments are much less than the barriers which exist in field space between the different local minima, and so we have no direct sensitivity to the presence of a possibly lower vacuum state. (A possible exception to this are magnetic monopoles which do probe the large field region.) If it is not the absolute minimization of the effective potential, what does determine the present vacuum state of the Universe? We argue here that it is determined cosmologically by the dynamical evolution of the Universe from the hot, dense phase which existed shortly after the big bang to the present[1,2].

At finite temperature, the free energy tends to be minimized in phases which contain light particles. Therefore, we expect that in general, at extremely high temperatures the thermodynamically favoured phase is the symmetric one which contains the most massless particles. Assuming that very near the big bang this phase is realized[3–5], one can follow the subsequent dynamical evolution to determine the present state. The present vacuum is properly determined by evolutionary, not merely energetic, considerations. This point is relevant for all proposed models of elementary particle interactions.

In what conditions might we convince ourselves that we live in a metastable vacuum (or, not finding these conditions, assure ourselves that we do not)? Of course, we must demand that the putative vacuum has an extremely long lifetime, and that small fluctuations around it describe known physics. In addition, it should have been realized in the dynamical evolution of the Universe, starting from the symmetric phase at extremely high temperatures.

First, consider the question of metastability. For a ϕ^4 scalar field theory in which the energy difference between the two minima is not too large, the decay rate of the 'false vacuum' (per unit volume per unit time) at zero temperature is[6]

$$\Gamma \simeq A \exp[-(32\pi^2/(3\varepsilon^3\lambda)] \quad (1)$$

where λ is the quartic self-coupling, ε is the ratio of the energy difference between the two minima to the barrier height between them, and A has dimensions of mass^6 and should be $0(\mu^4)$. Here μ^2 is the bare mass of the scalar field. Taking μ to be of the order of the grand scale ($\sim 10^{14}$ GeV), and the spatial volume to be that of the present observable Universe ($V \sim 10^{84}$ cm^3), the probability that an energetically less favourable minimum has decayed during the age of the Universe is exponentially <1 so long as

$$\varepsilon \ll 0.6\,\lambda^{-1/3} \quad (2)$$

a condition which is easily satisfied.

We now examine the second and third issues in the context of the simplest unified model of particle interactions, the minimal SU(5) theory[7].

At zero temperature the effective potential can be written as[8,9]:

$$V(h,A) = \frac{-\mu^2}{2}\operatorname{tr}A^2 + \frac{a}{4}(\operatorname{tr}A^2)^2 + \frac{b}{2}\operatorname{tr}A^4 - \frac{\nu^2}{2}h^\dagger h + \frac{\lambda}{4}(h^\dagger h)^2 + \alpha(h^\dagger h)\operatorname{tr}A^2 + \beta h^\dagger A^2 h \quad (3)$$

where the discrete symmetry $A \to -A$ has been imposed, h is an SU(5) vector **5** Higgs field, and A is an SU(5) adjoint **24** Higgs field (traceless hermitian matrix). To reproduce the standard (and thus far very successful) SU(5) phenomenology, there must be a local minimum where A acquires a large vacuum expectation value of the form,

$$\langle A\rangle = \begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ 0 & & & & -3/2 \end{pmatrix}\delta \quad (4)$$

This vacuum expectation value breaks SU(5) down to SU(3) × SU(2) × U(1).

The second stage of symmetry breaking involves the fifth component of h acquiring a vacuum expectation value much less than δ (the small vacuum expectation value $\langle h\rangle$ will slightly perturb $\langle A\rangle$ from the form given in equation (4); see refs 8–10). In addition, we must also ensure that the colour triplet component of h has a large mass $\simeq 0(\delta)$ as it can mediate proton decay. Both of these requirements are fulfilled if:

$$\beta < 0 \quad (5a)$$

$$-m_{\rm eff}^2 \equiv \nu^2 - 3\mu^2(10\alpha + 3\beta)/a' \approx 0 \quad (5b)$$

$$(\alpha + \beta/2) > -[\lambda(a+b)]^{1/2}/2 \quad (5c)$$

$$\mu^2, a', b > 0 \quad (5d)$$

$$\lambda > 9\beta^2/10b \quad (5e)$$

where $a' \equiv 15a + 7b$. Condition (5b) specifies that the SU(2) doublet component of h (the ordinary Weinberg–Salam doublet) has a small, negative bare mass. Quantitatively, $m_{\rm eff}$ must be about 13 orders of magnitude smaller than $\delta = (2\mu^2/a')^{1/2}$.

For our purpose, the crucial observation is that condition (5b), along with the others, can be satisfied with ν^2 large and positive. Looking back at equation (3), it is perhaps not surprising that in this case another very different type of vacuum might be energetically favourable: one in which $\langle h\rangle$ is large. Indeed, if

$$2\alpha\nu^2/\lambda > \mu^2 \quad (6a)$$

$$2(\alpha + 4\beta/5)\nu^2/\lambda > \mu^2 \quad (6b)$$

then the potential also has a local minimum at $\langle h_5\rangle^2 = \nu^2/\lambda$ and $\langle A\rangle = 0$. In this vacuum SU(5) has broken down to SU(4). When we compare the value of the effective potential at this local minimum with the usual one (when both exist), we find that for the following range of parameters:

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$$\mu^2, \lambda, a', b, -\beta > 0 \quad (7a)$$

$$\lambda > 9\beta^2/10b \quad (7b)$$

$$(\alpha + \beta/2) > -[\lambda(a+b)]^{1/2}/2 \quad (7c)$$

$$(5\alpha + 4\beta)(10\alpha + 3\beta) > 5\lambda a'/6 \quad (7d)$$

$$\nu^2 \approx 3\mu^2(10\alpha + 3\beta)/a' \quad (7e)$$

the usual vacuum corresponds to a local, but not a global, minimum.

From the point of view of microphysics, then, it is quite conceivable that our vacuum is metastable. We must now investigate the dynamical question, whether it is reasonable that during the course of its temperature evolution, the Universe could get 'hung up' in our vacuum in spite of its only being metastable. Basically, we must determine which of the two asymmetric vacua becomes lower than the fully symmetric vacuum first as the Universe cools down from very high temperatures. Although we have not investigated this question in quantitative detail, it seems that a 'hang up' can occur. Finite-temperature modifications to the zero-temperature potential tend to increase the free energy, and those due to the most strongly coupled fields are most important[3-5]. Suppose that the usual vacuum and the SU(4) vacuum discussed above have roughly the same energy at zero temperature. The high temperature state continuously related to the SU(4) vacuum will then be disfavoured if it feels stronger finite-temperature corrections. This will occur, for instance, if all the Higgs self-couplings are small and h, but not A, couples strongly to some fermion field.

It seems distinctly possible in simple unified models, that our present vacuum is only metastable, and that nevertheless, the Universe would have chosen to get 'hung up' in it. If this is the case, then without warning, a bubble of true vacuum could nucleate somewhere in the Universe and move outwards at the speed of light, and before we realized what swept by us our protons would decay away. There is no immediate cause for concern, however, because the lifetime of our metastable vacuum could easily be 10^{30} yr (or only 10^{10} yr for that matter).

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